

# Beyond Standard Model Physics from Drell Yan

## Processes at FNAL

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### Introduction

Truly successful grand unification theories (GUT)s have been eluding physicists since the first attempt to unite the strong, weak, gravitational and electromagnetic forces nearly a century ago. While no particular GUT seems likely as an explanation of nature, several have been shown to be consistent with experimental data. In particular, GUTs that involve making the  $SU(3) \times SU(2) \times U(1)$  group (which accurately describes the standard model (SM)) a subgroup of a larger symmetry group have been quite successful. Even GUT schemes involving more exotic group theoretical techniques have shown promise. One consequence of several of these theories is the existence of a new neutral gauge bosons, typically referred to as  $Z'$  bosons. Generally speaking, these new gauge bosons are similar to the SM  $Z$  (which we will denote as  $Z_0$  for clarity). Detection efforts for these new bosons are currently underway. One means of detecting a  $Z'$  is to examine the  $\frac{d\sigma}{dM}$  or the  $A_{FB}$  for Drell Yan dilepton pair

production from  $p\bar{p}$  collisions, i.e.  $p\bar{p} \rightarrow Z' + (X) \rightarrow \bar{l}l + (X)$ . In the following, I will attempt to explain the beyond SM physics principles these searches are trying to find.

## Results of GUTs and String theory inspired Unification schemes

### *Background*

Quantum field theory has shown us that from first principles, one can construct a lagrangian utilizing various gauge groups that will behave like nature. Modern interpretations of field theory use a covariant derivative lagrangian,

$$\mathcal{L}_{fermion} = \sum_f \bar{f} i \gamma^\mu \mathcal{D}_\mu f \quad (1)$$

with the covariant derivative  $\mathcal{D}_\mu$  being

$$\mathcal{D}_\mu = \partial_\mu - ig_1 \left( \frac{Y}{2} B_\mu \right)_Y - ig_2 \left( \frac{\tau^i}{2} W_\mu^i \right)_L - ig_3 \left( \frac{\lambda^a}{2} G_\mu^a \right)_Q \quad (2)$$

This lagrangian gives us a description of nature that we have come to believe from a phenomenological standpoint in the field of particle physics. The second term in  $\mathcal{L}$  is the  $U(1)$  symmetry piece from hypercharge. The third term is the  $SU(2)$  group piece, giving us charged and neutral current interactions,  $W^{+-}$ ,  $Z_0$  bosons. The last term is the  $SU(3)$  group term, giving us the quark gluon interactions (the color charge piece). The  $g_{1,2,3}$  factors in front of the terms in the covariant derivative are the coupling strengths of the various interactions. [6, 5]

To build a model, one only has to use the asymptotic freedom of the  $L, Q$  terms and the negative slope of the electromagnetic interaction at high mass scales to make the gauge forces

meet. The mass scale at which you get the couplings to meet is your unification energy.

### *GUTs*

The SM, which has been more or less gospel for decades, agrees with experimental data excellently. The  $SU(3) \times SU(2) \times U(1)$  symmetry group provides all the gauge symmetries needed to describe everything from electromagnetic interactions with photons to the apparent impossibility of free quarks. However, the SM has been labeled inadequate. In spite of its success, the SM does not unify all the forces. The SM also has a number of *a priori* parameters and *ad hoc* features (e.g. heirarchy, left handed charged currents, etc.) [7].

The simplest idea for leptons and quarks, is to put the particles into a larger symmetry group; make  $SU(3) \times SU(2) \times U(1)$  a subgroup of a larger group. One of the most successful schemes involves an  $SU(5)$  representation, which without inputs to the theory, gives fractional quantization of quark charge and even the neutrality of atoms. Typical GUTs unifying electroweak and strong forces attempt to calculate an energy scale at which the coupling strengths of the forces are (hopefully) linearly related.

However, even the  $SU(5)$  representation unification scheme seems inadequate. None of the  $SU(5)$  theories can easily accomodate gravitational interactions. There is a good reason; gravity's mass scale is huge, as the length scale associated with gravity is the Planck length,  $R_{Pl} = 10^{-33}$ cm. That means that the current tools for calculating unification theories cannot bring gravity and the electroweak forces together; the coupling strengths would not converge to the same place in coupling-mass scale space [5].

Furthermore, The  $SU(5)$  representation has been shown to be inconsistent with  $\sin^2(\theta_W)$  and the proton lifetime,  $\tau_p$ . [1, 2]

*Extra gauge bosons from  $E_6$*

Prior to the discovery of the top quark, the  $E_6$  representation was noted to be the next highest anomaly-free gauge group beyond  $SO(10)$ . This led to a large amount of study, and as with many theories, its novelty eventually wore off as the models for the most part allowed the  $b$  quark and  $\tau$  lepton fill in the representation for a “top-less” model (salacious name, if you ask me). As soon as it was generally accepted that the  $b$  and  $\tau$  were members of a third generation, these models were all but abandoned. However, it was shown in the early 80s [7, 8] that the  $E_6$  models might be usable afterall. Green and Schwarz showed in 1984 that string theory in 10 dimensions is anomaly-free as long as the gauge group used is  $E_8 \times E'_8$  or  $SO(32)$ .  $SO(32)$  does not give chiral fermions, so it doesn't fit with the SM.  $E_8 \times E'_8$ , on the other hand, can contain the SM in its unaltered form, since its fermions create a chiral representation. To create a field theory, one simply takes the limit of large string tension, and the string excitations are integrated out. The resultant representation is a 10D supergravity coupled to an  $E_8 \times E'_8$  gauge sector. To get to our world, we need to compactify the remaining six dimensions into another manifold. Several different schemes have arisen, since the technical details of orbifold geometry allow for many different compactification topologies [7].

Calabi-Yau compactification with an  $SU(3)$  holonomy takes our  $E_8 \rightarrow SU(3) \times E_6$ . The  $SU(3)$  links spin to the compactified space. The remaining  $E'_8$  only couples to the matter states of the  $E_6$  in gravitational interactions, and may prove to be the SUSY breaking sector not yet seen, and may provide a solution for dark matter. After compactification from 10

to 4, an unbroken  $N = 1$  SUSY is left. Depending on the topology of the compactification scheme, several different rank 5 or 6 subgroups  $S$  are left from the  $E_6$ ,

$$\text{Rank 5 } S = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta$$

$$\text{Rank 6 } S = SU(3)_C \times SU(2)_L \times U(1)^3$$

$$SU(3)_L \times SU(2)^2 \times U(1)^2$$

$$SU(3)^3$$

$$SU(4) \times SU(2) \times U(1)^2$$

$$SU(2) \times U(1) \times G(n, \forall n \geq 2)$$

...

Since  $SU(3)_C \times SU(2)_L \times U(1)_Y$  has rank 4, we clearly have at least one other neutral gauge boson in the rank 5 or 6 cases. It is also clear that further symmetry breaking is needed. One common solution is an effective Higgs field. This does the job nicely, allowing the rank 5 representation to break down into  $U(1)_Y$  as necessary [7].

### Model 1

Perhaps the simplest case is the following breakdown:  $E_6 \rightarrow SU(5) \times U(1) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta$ . Here, the compactification scheme has left a single  $U(1)_\eta$  symmetry. This symmetry leads to a single neutral gauge boson, a  $Z'$ . This single  $Z'$  is one of the most studied cases, because of ease of calculation [7].

## Model 2

*Rank 5: Jane's crazy mixed up Z's*

In the rank 5 case,  $E_6 \rightarrow SO(10) \times U(1)$ . The  $U(1)$  may be labelled as  $U(1)_\psi$ , and the  $SO(10)$  may further break down;  $SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi$ , the new  $U(1)$  being labelled  $U(1)_\chi$ . Now there are *two* extra  $U(1)$  symmetries that can each lead to gauge bosons. These two  $U(1)$ s can form an orthogonal pair of abelian generators, independent of the electric charge. The generators for this symmetry group are:

$$Z'_\psi = \frac{1}{\sqrt{2}} (1 \ 0 \ 0 \ 0 \ 0 \ 0) \quad (3)$$

$$Z'_\chi = \frac{1}{\sqrt{10}} (0 \ 1 \ 1 \ 1 \ 1 \ 1) \quad (4)$$

If at least one of the mass eigenstates of these generators is light enough to be detected, it can be expected that a superposition state of the two may be visible also. An intermediate  $Z'(\theta)$  is defined as an overlap of the two states;  $Z'_1(\theta) = Z'_\chi \cos(\theta) + Z'_\psi \sin(\theta)$ , with  $\theta$  the  $(\chi - \psi)$  mixing angle. Another state is easily defined;  $Z'_2(\theta) = Z'_\chi \cos(\theta) - Z'_\psi \sin \theta$ . These overlap states could be seen easily if the mass of one or both is light enough to allow it in Drell-Yan processes at colliders [8, 7].

## Model 3

*Three lepton families?:  $G_{MSM} \times U(1)_{L_i}$*

Traditionally, the SM has three lepton families;  $e, \mu, \tau$ . These families can each be described by the quantum number  $L_{e,\mu,\tau}$ , the lepton family number. Under minimal standard model, MSM, the interaction lagrangian,  $\mathcal{L}_{int}$  is invariant under the symmetries  $U(1)_{e,\mu,\tau}$ , as

there are no lepton family violating processes observed. A new basis can be defined;  $L_{i=1,2,3}$ .

The rules are as follows:

$$L_1 = L_e - L_\mu \quad (5)$$

$$L_2 = L_e - L_\tau \quad (6)$$

$$L_3 = L_\mu - L_\tau \quad (7)$$

$$[L_i, L_j]_{i \neq j} = 1; i, j \in [1, 2, 3] \quad (8)$$

$$\Psi = \alpha_\xi L_e + \beta_\xi L_\mu + \gamma_\xi L_\tau \quad (9)$$

Where  $\Psi$  is the representation of any lepton,  $\xi \in [1, 2, 3]$ . With this new representation comes a new  $U(1)_{L_{1,2,3}}$  symmetry. The new representation is written  $G_{MSM} \times U(1)_L$ . This means that there is a second neutral gauge boson in addition to the SM  $Z_0$  for each of the new lepton re-defined bases,  $i = 1, 2, 3$ . So, the new  $Z'$ 's are labelled  $Z'_1, Z'_2, Z'_3$  accordingly. If the gauge couplings  $g'_{1,2}$  are small enough, the  $Z'_{1,2}$  could be massless.  $g'_3$  can be set [10] by standard methods for this massless limit to be  $\leq 10^{-24}$ , so there is room for an observable. The massless  $Z'_3$  would couple to the  $\mu(g-2)$ , and thereby imply that the  $g'_3 \leq 10^{-5}$ , making an observable far more difficult since  $\mu(g-2)$  is so well known [9].

## Model 4

*Low Energy effective EW group:  $U(1)_L \neq U(1)_Y$*

Again, compactifying from  $E_8 \times E'_8$  to  $E_6 \times E'_8$ , what remains will become the SM fields and an  $E_8$  “shadow world” with only gravitational interactions, respectively. The  $E_6$  has

room for much variation in gauging schema. As usual, the  $E_6$  fermions are put into a **27**-plet with the usual set of particles;  $u, \bar{u}, d, \bar{d}, e^\pm, \nu_{e,\mu,\tau}, \dots$ . But, not breaking down into the usual groups first gives rise to even more particles; right handed  $\nu$ 's, isosinglet  $-\frac{1}{3}$  charge  $q, \bar{q}$ , 2 lepton isodoublets  $(\nu_E, E^-), (E^+, \bar{\nu}_E)$ , and a neutral isosinglet lepton  $N$ .

A particular maximal subgroup of  $E_6$  is  $SU(3)_C \times SU(3)_L \times SU(3)$ , where the **27** =  $(3, 3, 1)_q + (\bar{3}, 1, \bar{3})_{\bar{q}} + (1, \bar{3}, 3)_l$ . The lepton  $SU(3)$  can break down into an  $SU(2)$  and a  $U(1)$ . However, it is interesting to note that this  $U(1)$  is not the hypercharge of the SM;  $U(1)_L \neq U(1)_Y$ , with  $Q_{EM} = I_{3L} + \frac{Y}{2}$  since otherwise the quark charge would be zero! Since  $SU(3)_L$  must contribute to the electromagnetic charge, the low energy effective electroweak group must be at least  $SU(2)_L \times U(1)_L \times U(1)$ , meaning that there must be at least one extra  $Z'$  in the low energy theory [8].

## Model 5

### *W, Z mixing due to large G subgroup*

In the typical scheme,  $E_6 \rightarrow SU(2) \times U(1) \times G(n)$ , we can have  $W$  and  $Z$  mixing from the new  $G(n)$ . For simplicity, the  $Z$  mixing will be discussed and the extension to the  $W$  follows logically. If the  $G(n)$  has an extra neutral  $Z'$ , the derivation from the extra  $U(1)$  from Model 1 stands. The extension to the  $W$  follows naturally, if the extra  $G(n)$  also has a charged current vector boson.

The compactification from  $E_6$  down to our world can have even more neutral gauge bosons. The  $SO(10)$  symmetry group has rank 6, but the  $SU(5)$  group has rank 5. So, compactifying from  $SO(10) \rightarrow SU(5)$  picks up an extra neutral gauge boson;  $SU(5) \times U(1)_\lambda$ . Likewise,  $E_6$  has one higher rank than  $SO(10)$ , so going from  $E_6 \rightarrow SO(10)$  picks up another neutral gauge boson;  $SO(10) \times U(1)_H$ . The boson that arises from compactifying from  $E_6$



has an index  $H$  because it is an  $Z'_H$ , a Yang-Mills coupling  $Z'$ . This  $Z'$  should show up in Drell-Yan collisions also for  $\sqrt{s} \simeq 1.6 - 2.4\text{TeV}$  [11].

### Model 6

*Yet another kooky scheme,  $SO(6) \times SO(4)$*

Another superstring-inspired (read: drug-induced) compactification scheme involves the as-of-yet not mentioned  $SO(10) \rightarrow SO(6) \times SO(4)$  breakdown. The thought is that  $SO(6) \sim SU(4)$ .  $SU(4)$  readily breaks down into the required  $SU(3)_C \times U(1)_Y$  group. The remaining  $SO(4) \sim SU(2)_L \times SU(2)_R \rightarrow SU(2)_L \times U(1)_{R_1} \times U(1)_{R_2}$ . Thus, there are now two right handed neutral current bosons,  $Z'_{R_{1,2}}$  in addition to the usual SM  $Z_0$ . The subsequent mixing of the  $Z'$ s and  $Z_0$ s is expected to be detectable. Since there is no restriction on such interactions, it is also expected that the  $W^\pm$  will mix with the heavier of the two  $Z_{R_{1,2}}$ s [12, 7].

### Model 7

*SUSY partners*

It is assumed that the SUSY partners of the SM fermions,  $\tilde{f}$ , could make a large contribution to the  $Z'$  width. If the right and left handed sfermions are degenerate mass states, the resultant production width,  $\Gamma(Z' \rightarrow \tilde{f}_{L,R} \tilde{f}_{L,R})$  would decrease as much as 50%. This would be exceedingly easy to see in the Drell-Yan process at a  $p\bar{p}$  collider [7].

## Charge Asymmetry, Mass limits, $\frac{d\sigma}{dM}$ , and $A_{FB}$

In the previous section, motivation for the search for extra neutral gauge bosons was presented. Several models suppose the existence of the  $Z'$  boson(s), but how do these models'  $Z'$ s become evident in the context of a collider experiment? Each model can give

a prediction of either the front-back asymmetry ( $A_{FB}$ ), modified production/decay widths ( $\Gamma$ ), or the mass dependent differential cross section  $\frac{d\sigma}{dM}$  from Drell Yan events in  $p\bar{p}$  colliders. The calculations are highly model-specific, calculations will follow the model number stated previously. The Drell-Yan  $A_{FB}$  and  $\frac{d\sigma}{dM}$  have been measured numerous times in the context of SM verification. However, the high-mass limit of the  $A_{FB}$  and the  $\frac{d\sigma}{dM}$  has been shown recently to have some deviation from the SM prediction [13, 14]. This deviation points to a possible  $Z'$  signal. The several different models' predictions can apply, and determining which model fits the experimental anomaly will be the task for future experimenters.

#### *Rank 5 models and their $Z'$ s*

The most obvious production signal for the  $Z'$  at a hadron collider is the typical  $p\bar{p} \rightarrow Z' + (X) \rightarrow l\bar{l} + (X)$ . In the rank 5 scenarios, the only two that are easily studied are the single  $U(1)$  and the  $U(1)_\psi \times U(1)_\chi$ , as in model 1 and 2 respectively.

In model 1, the two states,  $Z'_{1,2}$  can be further broken into a total of four states, for the angle  $\theta = 0, -90^\circ, -\sin^{-1}(\sqrt{\frac{5}{8}}) \simeq -52.24^\circ, -\sin^{-1}(\sqrt{\frac{3}{8}}) \simeq 37.76^\circ$ , typically labelled in literature [7] as models  $\psi, \eta, \chi, I$  respectively. Each model's couplings are different, so the actual figures for the masses of the  $Z'$ s will differ correspondingly. However, generally speaking we can write the lagrangian coupling for these  $Z'$ s and the SM  $Z_0$ ,

$$\mathcal{L} = \dots + \frac{g}{c_W} \left[ (T_3 - \sin^2 \theta_W) Q Z_0 + \left( \frac{5}{3} \sin^2(\theta_W) \right)^{\frac{1}{2}} Q' Z'_{(\psi, \eta, \chi, I)} \right] \quad (10)$$

for  $Q$  being the quantum numbers coupling the  $Z_0$  to its charges.  $Q' \equiv Q_\psi \cos(\theta) + Q_\chi \sin(\theta)$ , in the resulting effective rank 5 model resulting from the two  $Z$ s mixing to form a single state. Assuming no  $W - Z$  mixing,  $M_W = \frac{gv}{2}$ , where  $v^2 = v_1^2 + v_2^2$ . One then writes

the  $Z_0 - Z'$  mass matrix:

$$\mathcal{M}^2 = \begin{pmatrix} M_{Z_0}^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{pmatrix} \quad (11)$$

$$\frac{\delta M^2}{M_{Z_0}} = 2 \left( \frac{5}{3} \sin^2(\theta_W) \right)^{\frac{1}{2}} \frac{1}{v} (Q'_1 v_1^2 - Q'_2 v_2^2) \quad (12)$$

$$\frac{M_{Z'_i}^2}{M_{Z_0}^2} = \frac{20}{3} \sin^2(\theta_W) \frac{1}{v^2} \sum_{i=1}^4 (Q'_i v_i)^2 \quad (13)$$

The mass matrix can be diagonalized with an orthogonal transformation about  $\varphi$  via

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{pmatrix} \begin{pmatrix} Z_0 \\ Z'_i \end{pmatrix} \quad (14)$$

here, the angle  $2\varphi \equiv \tan^{-1} \frac{2\delta M^2}{(M_{Z_0}^2 - M_{Z'_i}^2)}$ , which gives two mass eigenstates,  $M_{1,2}$ . If the constraint is added that  $M_{Z_0} - M_1 \geq 0$ , the  $M_2$  is calculated to be  $\leq 350\text{GeV}$ . Such a signal should be able to be detected from detailed  $\frac{d\sigma}{dM}$  measurements away from the  $Z_0$  pole [7, 14].

### *Rank 6 and other models*

In model 3, the idea of redefining the lepton families was introduced. If the remaining  $U(1)_{L_i}$  is broken spontaneously,  $Z'_i$  get masses via the Higgs mechanism. This implies that we can add a Higgs scalar field  $S_i$  in the SM, but not under local  $U(1)_{L_i}$ . The non-zero vev for  $S_i$  breaks  $U(1)_L$  giving the  $Z'_i$  masses;  $M_{Z'_i} = g'_i \langle S_i \rangle$ . The natural decay mode we would look for is the  $p\bar{p} \rightarrow Z'_{L_i} + (X) \rightarrow l\bar{l} + (X)$ . Since the  $Z'_3$  couples to the  $\mu$  (g-2) strongly, we can toss it out on experimental evidence. So we only have to contend with the  $i = 1, 2$   $Z'_{L_i}$ s. The  $Z'$  interaction lagrangian can be written out for the fermions based on

the neutral gauge boson interaction lagrangian;

$$\mathcal{L}_\gamma = eA_\mu (\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu + \bar{\tau}\gamma_\mu\tau) \quad (15)$$

$$\mathcal{L}_Z = \frac{g_2}{\sqrt{1-x}} Z_\mu \left[ \left(x - \frac{1}{2}\right) \frac{\mathcal{L}_L}{eA_\mu} + x \frac{\mathcal{L}_R}{eA_\mu} \right] \quad (16)$$

with  $x \equiv \sin^2(\theta_W)$  and  $g_2$  the SM  $SU(2)$  coupling constant. From the prior definitions of our new lepton bases, the new lagrangians are:

$$\mathcal{L}_{Z'_1} = g'_1 Z'_{1\mu} (\bar{e}\gamma_\mu e - \bar{\mu}\gamma_\mu\mu) \quad (17)$$

$$\mathcal{L}_{Z'_2} = g'_2 Z'_{1\mu} (\bar{e}\gamma_\mu e - \bar{\tau}\gamma_\mu\tau) \quad (18)$$

The front back asymmetry is defined as

$$A_{FB} = \int_0^{\frac{\pi}{2}} \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{\sigma} \quad (19)$$

$$= \frac{3}{4} \left( \frac{G_{LL}^2 + G_{RR}^2 - 2G_{LR}^2}{G_{LL}^2 + G_{RR}^2 + 2G_{LR}^2} \right) \quad (20)$$

$$G_{LL} = \frac{e^2}{s} + \frac{g_2^2}{1-x} \frac{1}{s - M_{Z_0}^2} \left(x - \frac{1}{2}\right)^2 - \frac{g_1'^2}{s - M_{Z'_1}^2} \quad (21)$$

$$G_{RR} = \frac{e^2}{s} + \frac{g_2^2}{1-x} \frac{1}{s - M_{Z_0}^2} x^2 - \frac{g_1'^2}{s - M_{Z'_1}^2} \quad (22)$$

$$G_{LR} = \frac{e^2}{s} + \frac{g_2^2}{1-x} \frac{1}{s - M_{Z_0}^2} x \left(x - \frac{1}{2}\right) - \frac{g_1'^2}{s - M_{Z'_1}^2} \quad (23)$$

for  $s$  the incident 4-momentum squared. Similarly, the cross sections can be defined;

$$R = \frac{\sigma(e^\pm \rightarrow l^\pm)}{\sigma_{QED}(e^\pm \rightarrow l^\pm)} ; [l = \mu, \tau] \quad (24)$$

$$= \frac{s^2}{4e^4} [G_{LL}^2 + G_{RR}^2 + 2G_{LR}^2] \quad (25)$$

$$(26)$$

So, for different values of  $g'_{1,2}$  relative to the SM  $g_2$ , limits can be set on the maximum  $M_{Z'}$ , and the shape of the resultant  $A_{FB}$  expected in Drell Yan  $p\bar{p}$  dilepton interactions [8].

In model 4, only one extra  $Z'$  is expected. In this scenario, the  $SU(3)^3$  breaks down into  $SU(3)_C \times SU(3)_L \times SU(2) \times U(1)$ . This leads to a 3-fold ambiguity over which remains, so we must choose which two leptons of  $\bar{u}, \bar{d}, \bar{h}$  will form the  $SU(2)_L$  doublet we want to see.

We set the following,

$$Q_{EM} = \left( 0 \ 1 \ \frac{-1}{3} \ \frac{-1}{3} \ \frac{-1}{3} \ 1 \ 0 \right) \quad (27)$$

$$= I_{3L} + \frac{1}{2}Y_L + G \quad (28)$$

$$I_{3L} = \left( 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ \frac{-1}{2} \right) \quad (29)$$

$$Y_L = \left( \frac{-1}{\sqrt{3}} \ \frac{-1}{3} \ \frac{-1}{3} \ \frac{-1}{3} \ \frac{1}{3} \ 0 \ 0 \right) \quad (30)$$

Now, we can write the cross section and the width for  $p\bar{p} \rightarrow Z' + (X) \rightarrow l\bar{l} + (X)$ . The cross section per unit rapidity is defined in the following manner:

$$\frac{d\sigma}{dy} = \frac{4\pi^2 x_1 x_2}{3M_{Z'}^3} \sum_{ij} f_i^p(x_1) f_j^{\bar{p}}(x_2) \Gamma_{ij} \quad (31)$$

where  $f$  is the structure function for the labelled hadron,  $x_{1,2}$  are the momentum fractions, and  $\Gamma_{ij}$  is the partial width of the  $Z' \rightarrow \bar{l}$ . Note that the definition of “lepton” here is both  $l$  and  $q$ . The partial width is defined as

$$\Gamma_{(Z' \rightarrow \bar{l})} = G_F \frac{M_{Z_0}^2}{\sqrt{2}} \frac{M_{Z'}}{4\pi} \frac{2\vartheta \sin^2(\theta_W)}{3} \left( [Q_L(l)]^2 + [Q_R(\bar{l})]^2 \right) \quad (32)$$

where the new coupling is given by  $2\vartheta = (g^2 + g'^2)^{\frac{1}{2}} \left(\frac{5}{3}\right)^{\frac{1}{2}} \sin(\theta_W)$ , and the  $Q_{L,R}$  are the quantum numbers corresponding to the charges that the  $Z'$  couples to. From the partial width and cross sections, the  $A_{FB}$  can be calculated directly. Though, in the interest of space, this particular calculation will be omitted. Suffice it to say, the  $A_{FB}$  would give us a window to look at the high-mass behavior of the couplings. The  $\Gamma_{ij}$  has free couplings, so one can fit to the measured  $A_{FB}$ - effectively “tuning” the coupling. For  $2\vartheta = 90^\circ$ , the  $M_{Z'} \geq 300\text{GeV}/c^2$  for  $\sqrt{s} = 2\text{TeV}$ . Similarly, given a coupling of  $2\vartheta = 0^\circ$ ,  $M_{Z'} \geq 200\text{GeV}$  [8, 15]. Thus, given the recent experimental data from CDF [14], further study of the Drell Yan  $A_{FB}$  is certainly warranted.

In model 5,  $W$  and  $Z$  mixing is allowed to occur in an  $SU(2) \times U(1) \times G(n)$  representation. Since we don't see  $W, Z$  mixing in the SM, we need to have the mixing in the extra  $G(n)$ . Writing the mass matrix again, we see

$$\mathcal{M}^2 = \begin{pmatrix} M_{Z_0}^2 & b \\ b & M_{Z_{G(n)}}^2 \end{pmatrix} \quad (33)$$

where the  $M_{Z_{G(n)}}$  is the mass of the new mixing  $Z'$ . The  $b$  is simply an arbitrary “mixing” mass. Two orthogonal states can be generated through a transformation exactly like equation 14. The resulting mass eigenvectors are

$$Z'_1 = Z_0 \cos(\varphi) + Z' \sin(\varphi) \quad (34)$$

$$Z'_2 = -Z_0 \sin(\varphi) + Z' \cos(\varphi) \quad (35)$$

To set constraints on the masses, the couplings can be manipulated, just as in other models. If one makes the supposition  $M_{Z'1}^2 < M_{Z_0} \leq M_{Z_{G(n)}}^2 \leq M_{Z'2}^2$ , the following constraint can be formed;

$$\tan^2(\theta) = \frac{M_{Z_0}^2 - M_{Z'1}^2}{M_{Z'2}^2 - M_{Z_0}^2} \quad (36)$$

Given the SM  $Z_0 = 93.8\text{GeV}$ , for  $M_{Z'1} = 91.9\text{GeV}$ , ( $1 \sigma$  below  $Z_0$  at the time of this analysis [11],  $\tan^2(\theta) \Rightarrow 500\text{GeV} \leq M_{Z'} \leq 1\text{TeV}$ . This again would show up in the differential cross section and the  $A_{FB}$ ;

$$\frac{d\sigma}{d\cos(\theta^*)} \propto (L_q^2 L_l^2 + R_q^2 R_l^2) (1 + \cos(\theta^*))^2 + (L_q^2 R_l^2 + R_q^2 L_l^2) (1 - \cos(\theta^*))^2 \quad (37)$$

for  $\theta^*$  the angle between the quark and the lepton [11].

Model 6 incorporated two additional  $Z'$ s. In that formalism, the  $SO(10)$  reduces to  $SU(3)_Q \times SU(2)_L \times U(1)_L \times U(1)_a \times U(1)_b$ . The new lagrangian is then written and parametrized in a new mass matrix;

$$\mathcal{L} = \partial^\mu + i(g_2 T \cdot W^\mu + g_a T_a B_a^\mu + g_b T_b B_b^{m\mu}) ; [SU(5) \times U(1)^2] \quad (38)$$

$$\mathcal{M}^2 = M_0^2 \mu^2 \quad (39)$$

$$M_0 = Z_0 = \frac{1}{2}(g_2^2 + g'^2)^{\frac{1}{2}} \nu_1 ; g' = \left(\frac{3}{5}\right)^{\frac{1}{2}} g_1 \quad (40)$$

This, of course, complicates the diagonalization,

$$\mu^2 = \begin{pmatrix} \cos^2(\theta) & \sin(\theta) \cos(\theta) & \frac{3\hat{g}\sqrt{10}}{\cos}(\theta) \\ -3 \sin(\theta) \cos(\theta) & \sin^2(\theta) & -\frac{3\hat{g}}{\sqrt{10}} \sin(\theta) \\ \frac{3\hat{g}}{\sqrt{10}} \cos(\theta) & -\frac{3\hat{g}}{\sqrt{10}} \sin(\theta) & \hat{g}^2 \left(\frac{9}{10} + \frac{5}{2} \frac{\nu_2^2}{\nu_1^2}\right) \end{pmatrix} \quad (41)$$

with  $\hat{g} \equiv \frac{g_{a,b}}{\sqrt{g_2^2 + g'^2}}$ .

Now the new mass matrix takes shape:

$$\mathcal{M}^2 = \frac{\nu_1^2}{4} \begin{pmatrix} g_2^2 & 0 & -g_2 g_{B-L} \\ 0 & g_R^2 & -g_R g_{B-L} R \\ -g_2 g_{B-L} & g_R g_{B-L} & g_{B-L} \left(\sqrt{\frac{2}{3}} \frac{\nu_2^2}{\nu_1^2}\right) \end{pmatrix} \quad (42)$$

Now, following the standard techniques, we can calculate the contribution to the  $\frac{d\sigma}{dM}$  and the  $A_{FB}$ . The calculation for the Drell Yan production of a  $Z'$  at a FNAL is given by

$$\left. \frac{d\sigma}{dy} \right|_{y=0} \left( p\bar{p} \rightarrow Z' \rightarrow \mu^+ + \mu^- \right)_{\sqrt{s}=1.9\text{TeV}} \simeq 7 \times 10^{-37} \text{cm}^2 = 0.7 \text{pb} \quad (43)$$



and thus, the cross section is large enough that in the famed upcoming Run2b, we should be able to detect any such  $Z'$  [12].

## Conclusion

Clearly, there is room for beyond SM physics in the high mass limit of both the Drell Yan  $A_{FB}$  and the  $\frac{d\sigma}{dM}$ . Since the cross sections are relatively large, and the signatures will be quite glaringly non-SM, this particular set of tools and models should prove to be an interesting analysis. The plethora of theoretical models supporting non-SM neutral gauge bosons is astonishing. Many models are self-consistent, and do not conflict *per se* with the current data available. Further study will probably unequivocally rule out or support some of the models examined in this paper, as the  $A_{FB}$  and  $\frac{d\sigma}{dM}$  for Drell Yan events should be a good measure of the models' validity.

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